Strategies for guiding student problem solving in PMA (Physics, Mathematics, and Astronomy)

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Objectives: This document will help you…

- Recognize the importance of active learning for students as they become independent problem-solvers
- Articulate your “expert” problem solving process so that you can use it to help guide students
- Combine problem solving guidance with active learning in strategies you can use when you are (a) working at the board with a larger section/group of students and (b) guiding smaller groups of students collaborating on problems.

Part 1: Intro to Active Learning

Big idea: Although most of your experience learning physics, math, and astronomy has likely been through lecture in class and doing problems outside of class, we now have a lot of evidence showing that this is not the most effective way to learn. Once you learn how to build active learning into the classes and recitations you teach, it will benefit students much more. This approach can be more enjoyable and rewarding for you, too, since the focus is on students’ thinking and engagement, rather than on you alone.

Definition of active learning: Structured in-class activities, led by an instructor or TA, which require students to process and apply information in a variety of ways, such as:

- Answering questions
- Completing worksheets or exercises
- Discussing and solving problems with fellow students

Sources/further reading (links to full text articles are included)

- Carl E. Wieman, Large-scale comparison of science teaching methods sends clear message, PNAS 2014;111:8319-8320.
- Scott Freeman et al., Active learning increases student performance in science, engineering, and mathematics, PNAS 2014; 111:8410-8415.
Part 2: Developing Problem Solving Expertise

Big idea: The faculty of the Caltech Division of Physics, Math, and Astronomy (PMA) value and seek to help undergraduates develop independent problem-solving abilities. This kind of “expertise” differs starkly from how “novice” (beginning or early undergraduate) students often approach problem solving. In addition, expert instructors/TAs can forget what it’s like to be a novice, since their problem-solving processes are so familiar and automatic. We call this forgetting “expert amnesia,” and it sometimes leads instructors/TAs to say things like “it is obvious that…” or “after a few trivial steps…”. Experts might even leave out of their explanations the very parts of problem solving that are most difficult and important for students to learn and practice. Once you bring attention to your own expert problem-solving process and articulate fully all of the important steps, you will have a powerful teaching tool to use with students.

Sources/further reading (links to full text articles are included)

- How People Learn, Ch. 2, How Experts Differ from Novices, National Academies, 2000, pp.31-50.
- Keith Weber, Students’ difficulties with proof, Mathematical Association of America

Exercise

Write down all of the steps you typically use when solving a new problem/proof in your field. Some of the steps might be implicit, or might come so easily to you now that you don’t have to write them down because you do them very quickly in your head. It might help you expand on your list of steps to think of a problem that’s challenging for you now. When you work on a difficult problem and get stuck, what are some steps you might take to work through the challenge and find a solution?

It’s important to articulate YOUR problem-solving process and discuss with fellow instructors/TAs, so that it will be easy to share with students! However, if you need some ideas to get started or want to compare your process with what other experts have found, here are a few examples:

- George Polya’s problem-solving method, as articulated in How to Solve It (1945)
- Donald E. Simanek’s Problem Solving in Physics (2004)
Part 3: Guiding Student Problem Solving Using Your Expert Process

Big idea: Now that you’ve articulated your expert problem solving process, including details and important steps that beginning students might leave out, use it to guide students in all phases of the teaching and learning process: to structure your explanations and examples, to give students feedback, and to prompt student thinking. By reinforcing these expert strategies in active learning modes, you help students build robust expert processes of their own and move toward independent problem solving.

Below are strategies to use when (a) you’re the instructor or TA “at the board” (leading a medium-to-large recitation section or group of students), and (b) you’re working with individuals or small groups of students, such as during office hours or during a recitation built on student collaboration or group work.

(a) When you’re the instructor/TA “at the board”

1. **Structure your material:** just like having a structure for problem solving helps students learn, having a structure for your recitation session overall helps them organize their learning. Make a plan for the day and share it with students, e.g., a brief outline at top left of the board; a list of examples for the day and what key concepts they contain.

2. **Name the problem-solving steps whenever you can:** When you’re working through a problem at the board, highlight the generalizable steps in the problem, e.g., “Like we always do, first we have to make sure we fully understand the problem by defining terms and drawing a diagram.” Naming steps prevents students from seeing each example problem as a singular case, and rather as a chance to practice a generalizable strategy and connect problems to each other to form categories they can more readily use in the future.

3. **Get students involved:** Even when you’re at the board, you can implement active learning! For any example problem, your expert problem-solving process provides you with excellent prompts to launch students’ active thinking during class/recitation. Every step can be a question stem: E.g.:
   - What are the important terms and variables in this problem?
   - What should go in our diagram?
   - What units are involved here?
   - What else do we know about this problem (e.g., from theorems, laws, or equations we have derived in the past)?
   - When we get stuck, some other strategies we’ve talked about are (e.g., working backwards, solving a simpler case, …). What could we try next in this problem?
Helpful strategies for active learning with a large group:

☑ Ask open-ended questions that have multiple ways to answer. E.g., Why, How, What (does this mean, are you thinking about, etc.). These types of questions get students thinking in ways that help them become expert problem solvers. They also lower the perceived risk or barrier to answering, because you’re not looking for just one right answer. Avoid closed questions (one right answer/ “yes/no”).

☑ Acknowledge participation positively – say “thank you” when students speak up in class and mention what’s good about their thinking. E.g., “Thanks for that idea, Zora. You clarified the problem helpfully.” When you need to correct them, you’ll do so building on a base of respect, e.g. “That’s a common way to think about the problem and at first glance it might seem like it will work. Can you think of a reason it might not / here’s why it’s not correct in this case?”

☑ Increase your wait time. When you ask a question, students have to process the question, make sure they understand it, and then think about how to answer. Some will do this faster than others, but it takes time for everyone. Commit to waiting a set amount of time before calling on anyone. Let students know what you’re doing, e.g., “I’m going to give you a little time to think about it first, then see what ideas you have” and then you count slowly / silently to 10 or 20.

☑ Encourage more / different students to participate. You don’t always have to call on the same people who raise their hands first. Announce a strategy and then stick to it, and others will get involved. E.g.:  
  - “I’m going to wait until several people have something to share.”
  - “I’d like to hear from the back (front/left/right/middle) of the room this time.”
  - “Anwar, how could we begin this problem?”

☑ Use Think-Pair-Share. Follow these steps to get EVERYONE actively learning and make it easier to solicit answers and contributions from the class:

  - THINK: Give the class a moment to think—from 10 seconds to a few minutes. “Here’s a question for you. I’m going to give you X seconds/minutes to think about it on your own and write down your ideas.” In general, give less time than you think they need; it doesn’t have to take a lot of time to work well.

  - PAIR: Have each student discuss their ideas with a neighbor. Ask them to compare reasoning, convince their partner, come up with as many hypotheses or approaches as they can, etc. Walk around and listen to their discussions for instant feedback and ideas about what to do next. Stop them before they are done—interrupting while they still have more to say is good.

  - SHARE: Ask for ideas from a few pairs. If you listened to their conversations, you can ask specific pairs to explain ideas or approaches that you think would benefit the rest of the class. Anyone you call on is likely to be more confident and less hesitant, since they’re speaking for their group and had adequate time to hone their ideas with a neighbor.
(b) When you’re working with small groups of students

1. Structuring small group work.
   If your class or recitation is going to be built around having students work on problems in groups for most or much of the time, here are a few pointers. Many of these work well for office hours, but less formally.

   - **Student groups of 3-4 works best.** That’s small enough for everyone to participate, but big enough for diverse perspectives and ideas.

   - **Plan to mix up the groups regularly** (every ~2 weeks). Random groups at first is fine. Then, try to mix up groups to include different backgrounds, styles, personalities. You can do this by having students count off (1, 2, 3, 4, 5, 1, 2, 3…) and then group by shared number, or post groups you’ve randomly assigned beforehand. Changing groups allows you to intervene if there are any groups that really don’t get along, and it benefits everyone’s thinking to work with different perspectives over the course of the term.

   - **Make sure students change roles within their group often.** Don’t let the same person always “hold the chalk” (if they’re working at a board), speak for the group (e.g., when you circulate to check in with them), etc. Be direct about requesting that they change roles when needed, e.g., “Dana, I notice you’ve been at the board for a while today. Let’s have Lakshmana take over.”

2. Your role: circulate and facilitate to help students get unstuck
   When students are collaborating and working together on problems, you still have an active role. As instructor or TA, actively walk around. Spend a little time with each group. Listen to where they are, or ask for an update.

   - **Check for productive but not excessive struggle.** Group work in class is meant to be different than homework, because you are there to guide and support students. It’s good to intervene when a group is stuck.

   - **Acknowledge what the group is doing well.** Sometimes they don’t know and feel lost about everything. Let them know what’s correct so far.

   - **Use your expert problem-solving process with groups.** Just like you would with the whole class (see p. 3-4), use prompts based on steps in your problem-solving process to help the group think through next steps. With a small group, you can ask specific students by name to chime in with their thoughts.

3. Bring everyone together when needed
   Even in a group collaboration-based class/recitation, it can be useful to convene everyone back together at key moments.

   - **At the beginning:** starting the class or office hours session off with an outline or overview helps get everyone oriented and ready.
• When you notice common challenges or mistakes, that’s a great time to ask for everyone’s attention to clarify or correct a shared misconception. Then they can go back to their work.

• When you want them to move on: groups work at different paces; it’s ok if some don’t finish everything, as long as there’s an opportunity to understand the problem. When a few groups are nearly finished, that can be a good time to bring everyone together to go over the main points of the problem and answer any questions.

BONUS: Encouraging students to ASK QUESTIONS

No matter what format you use in your class, recitation, or office hours, you can help make it more likely that students will ask their questions. Here are some strategies:

• Apply the same techniques as above!
  ✓ Increase your wait time: e.g., “What questions do you have about X?” silently count to 10 or 20 while waiting patiently.
  ✓ Do a think-pair-share for questions: e.g., “I’m going to give you 30 seconds to write down any questions or topics that are unclear, and then compare your list with a neighbor.”
  ✓ Ask for questions using an open-ended question: e.g., “What would you like me to clarify about X?” (instead of “Do you have any questions?”, which is itself a closed question with a yes/no answer.)

• How you answer questions matters:
  ✓ Thank students for asking questions! Communicate that you want and value their questions, and that their questions are normal and common: e.g., “Thanks for asking that, Sashi. I think others probably have that same question.”
  ✓ If you’re not sure of the answer, it can be nerve-wracking. Remember, you have a lot of expertise to share. Here’s some advice from expert teachers:

    Think out loud. Draw on your now-familiar expert problem-solving process and name the steps in your thinking for students. This models good problem-solving, even when you don’t know the answer: e.g., “I haven’t seen this case before, but here’s how I would start to think about it. First…”

    Bail out if you need to: e.g., if it gets too messy or gets in the way of other goals. There’s no prize for slogging through a problem gone bad when you have other important things to do that day: e.g., “This turns out to be pretty complicated. Because we have some other important examples to get to today, I’m going to look at this later and get back to you with an answer.”